AIAA JOURNAL
Vol. 30, No. 2, February 1992

Impingement Tones of Large Aspect Ratio Supersonic Rectangular Jets

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When a supersonic rectangular jet of large aspect ratio is directed normal on a wall, numerous tones, sometimes as many as 20 or more, are generated. These tones are produced by feedback loops consisting of downstream propagating instability waves of the jet flow and upstream propagating acoustic waves from the wall to the nozzle lip. The frequency structure of those tones is analyzed. It is found that there are only two basic tones. The lower frequency basic tone is associated with a symmetric instability feedback mode of the jet. The higher frequency basic tone is associated with an antisymmetric instability feedback mode. All of the other tones are combination tones of the two basic frequencies and their harmonics. The feedback acoustic waves are investigated following a recent suggestion that they are formed by the upstream propagating neutral acoustic wave modes of the jet flow. These wave modes, just as the familiar Kelvin-Helmholtz instability waves, are supported by the mean flow of the jet and can be calculated in the same way. An analysis of these waves using a vortex sheet jet model is carried out. The calculated dispersion relations indicate that these waves can be divided into the symmetric and the antisymmetric families. Each family of waves consists of infinitely many modes. Each mode is, however, confined to a narrow band of frequencies. The frequencies of the first mode of each family are low enough to match those of the jet instability waves to form feedback loops. All of the recently measured basic tone frequencies are found to fall within (or very close to) the calculated allowable frequency bands over a wide range of jet operating conditions. This provides strong support for the validity of the most recently proposed feedback model.

I. Introduction

EARLY investigators, e.g., Wagner[1] and Neuwerth,[2,3] found that strong tones were emitted when a high subsonic or supersonic jet was directed normal on a wall. It was observed that when the distance between the nozzle exit to the wall was gradually increased the frequency of an impingement tone often changed abruptly. This frequency jump phenomenon, sometimes referred to as staging, is known to be characteristic of systems driven by acoustic feedback. It is well known that jets are highly unstable. These same instability waves are the energy source of the feedback loop. Figure 1 is a schematic diagram of the various links of the feedback loop of a subsonic impinging jet. At the nozzle exit acoustic disturbances excite the large-scale instability waves of the jet flow. The instability waves grow in amplitude as they propagate toward the wall. Upon impinging on the wall, acoustic disturbances are generated. Some of the acoustic disturbances propagate upstream toward the nozzle. On reaching the nozzle exit where the shear layer of the jet is thin and most receptive to external excitation, they excite and generate further instability waves. In this way the feedback loop is closed.

The feedback idea was well recognized by Wagner[1] and Neuwerth.[2] By recording the motion of his round impinging subsonic jets on high-speed movies, Neuwerth was able to clearly identify the feedback acoustic disturbances inside the jet column. Since the feedback loop is closed, the time taken for disturbances to go around the loop once must be equal to an integral multiple of the period of oscillations or the period of the impingement tone. Let c_i and c_a be the phase velocities of the downstream propagating instability waves and upstream propagating acoustic waves, respectively, L the distance of separation between the nozzle exit and the wall, and f_n the impingement tone frequency. Then the feedback condition requires f_n to be given by

$$f_n = \frac{n}{L(1/c_i + 1/c_a)}, \qquad n = 1, 2, 3, \ldots \tag{1}$$

In deriving Eq. (1), it has been implicitly assumed that the time involved in the reflection and excitation processes of the feedback loop at the wall and at the nozzle exit is negligible. The staging phenomenon alluded to before is the result of an abrupt change in the integer value n as the nozzle-to-wall distance increases.

Since the pioneering work of Wagner and Neuwerth, the impingement tone phenomenon has been studied by a number of investigators.[4–7] Ho and Nossier,[4,5] in particular, examined the feedback loop and measured the acoustic field associated with the tones for subsonic impinging jets. Krothapalli[8] appeared to be the first to investigate the tones of supersonic rectangular jets. A more recent comprehensive experimental study of these jets was carried out by Norum.[9] Despite these various studies, a precise understanding of the feedback path is still lacking. For instance, in their work on subsonic jets Ho and Nossier[4] suggested that the feedback acoustic waves propagated from the wall to the nozzle exit outside the jet. This differed from the earlier proposal of Wagner[1] and Neuwerth,[2] who believed and presented evidence to the effect that these waves actually propagated upstream inside the jet. This point

Received Aug. 16, 1990; presented as Paper 90-3984 at the AIAA 13th Aeroacoustics Conference, Tallahassee, FL, Oct. 22–24, 1990; revision received April 6, 1991; accepted for publication April 16 1991.

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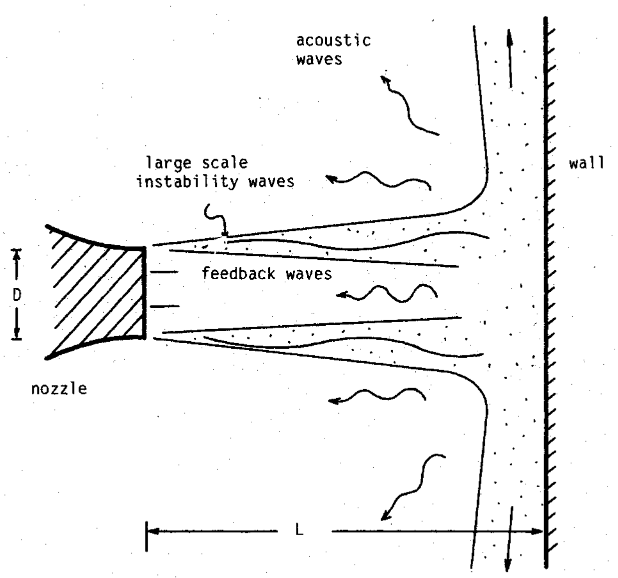

Fig. 1 Schematic diagram of the feedback loop of a subsonic impinging jet.

was discussed at some length in a recent article by Tam and Ahuja.[10] In this most recent work, it was proposed that the feedback was achieved by a family of upstream propagating (neutral) acoustic waves of the jet flow. These waves can exist inside a subsonic jet but attached themselves to the outside of a supersonic jet. The characteristics and structures of this family of wave modes are determined by the mean flow of the jet in much the same way as those of the Kelvin-Helmholtz instability waves. Based on the calculated dispersion relations and eigenfunctions of these neutral wave modes, Tam and Ahuja were able to predict the averaged impingement tone frequency at different jet Mach numbers and also provide, for the first time, an explanation of why no tones were observed (cold jets) when the jet Mach number was less than 0.6. The predicted impingement tone Strouhal number agreed well with all the available measurements. In addition, after examining all existing optical data, they noticed that there is seemingly a fundamental difference between the behavior of supersonic and subsonic impinging jets. For subsonic round jets, the instability waves and the feedback acoustic wave modes are invariably axisymmetric. However, for supersonic jets, both axisymmetric and helical (flapping) modes have been observed. A simple explanation of this rather unusual Mach number dependence is possible if, indeed, the feedback is accomplished by the upstream propagating neutral waves.[10] It turns out that the allowable frequencies of the helical upstream propagating neutral wave modes are restricted to narrow bands. These allowable frequencies are too high to match those of the instability waves of the jet at subsonic Mach numbers so that no helical feedback loop is possible. As jet Mach number increases, the frequencies of the allowable frequency band decrease. At supersonic Mach number, frequency matching with the instability waves becomes possible so that stable helical feedback loop and hence impingement tones can be maintained.

The primary objectives of this paper are to obtain a better understanding of the tone frequency structure of impinging supersonic rectangular jets and to examine whether the feedback path is consistent with the proposal of Tam and Ahuja.[10] In the recent work of Norum,[9] it was shown that a supersonic rectangular jet of large aspect ratio (aspect ratio greater than 4) generated a multitude of tones. As many as 20 to 30 tones, not harmonics of each other, were observed. A simple explanation of this multiplicity of tones and possible relationships of the frequencies of these tones are investigated. The impingement tones are generated by acoustic feedback so that the tone frequency must satisfy Eq. (1). However, for tone frequency prediction purposes, this formula is not helpful since the appropriate value of integer n to be used is not known a priori. It will be shown later that if the proposition that the feedback is accomplished by the upstream propagating neutral wave modes of the jet[10] is accepted, it is possible to predict the tone frequencies to within a narrow frequency band over a fairly wide range of jet Mach numbers.

II. Impingement Tones of Supersonic Rectangular Jets

In a recent investigation of the noise of impinging supersonic rectangular jets of large aspect ratio, Norum[9] found that they were rich in tones. In many cases as many as 20 or more tones were recorded. Not all these tones are harmonics of one another. Typical examples are given in the spectra data of Figs. 2–4. The existence of such multitude of tones was never reported in earlier studies of supersonic round jets. Even in the rectangular jet experiment of Krothapalli,[8] because of the small jet size and the limited frequency band used, the richness in tones was not specifically emphasized. Since the publication of Ref. 9, a careful analysis of all of the measured data (including many sets of unpublished data) has been carried out. It was found that all of the tones are combination tones of two basic frequencies. The lower of the two basic frequencies is invariably associated with a purely symmetric (varicose) feedback instability wave mode of the jet. The higher frequency is associated with a purely antisymmetric (sinuous) or flapping feedback instability wave mode of the jet. The reason that the symmetric oscillations have lower frequency will become clear later. Norum[9] was the first to provide clear experimental evidence of the simultaneous existence of these two modes of oscillations in impinging supersonic rectangular jets. Figure 5 shows the symmetric feedback oscillations of a Mach 1.29 jet at a frequency of 3.4 kHz. The jet issued from a convergent nozzle. Figure 6 shows the antisymmetric feedback oscillations of the Mach 1.29 jet at a frequency of 6.3 kHz. It is to be emphasized that the jet undergoes both symmetric and antisymmetric instabilities simultaneously. Figures 5 and 6 were obtained by exposing the film to a stroboscopic light source activated by the microphone signal filtered around the frequency of the tone. The multiple exposure enhances the visibility of the motion of the jet at the strobing frequency.

In the measured spectra, the two basic frequencies are generally those of the two tones with the highest amplitudes. These amplitudes are substantial so that strong nonlinear interaction between the two basic unstable modes of the jet takes place. The nonlinear interaction leads to jet oscillations and tones at frequencies that are combinations of the two basic frequencies and their harmonics. Figure 2 shows the noise spectrum of a supersonic impinging rectangular jet at a fully expanded Mach number of 1.29. The breadth and width of the nozzle are 2.013 and 0.475 in., respectively, giving an aspect ratio of 4.24. The nozzle-to-wall separation distance is 5.25 in. The microphone was mounted near the nozzle slightly upstream of the nozzle exit plane. As can be seen, the spectrum is completely dominated by tones. The two highest tones are the two basic frequencies. The one labeled s is that of the symmetric feedback mode. The one labeled a is the tone of the antisymmetric mode. The frequencies of 25 peaks identified as 1–25 are combination tones with frequencies equal to the sums and differences of the two fundamental tones and their harmonics. For instance, the frequency of tone 1 is the difference $a - s$. The frequency of tone 2 is the second harmonic with frequency $2s$. The frequency of tone 4 is equal to the sum $s + a$ and so on. The complete listing is given in the legend of Fig. 2. Figure 3 provides another illustration of the tonal structure at the same jet Mach number but at a closer nozzle-to-wall distance of 3.15 in. In this figure 20 tones other than the two basic frequencies are identified as 1–20. Under this particular jet operating condition, the difference of the two basic frequencies is smaller than the symmetric mode frequency so that the low frequency

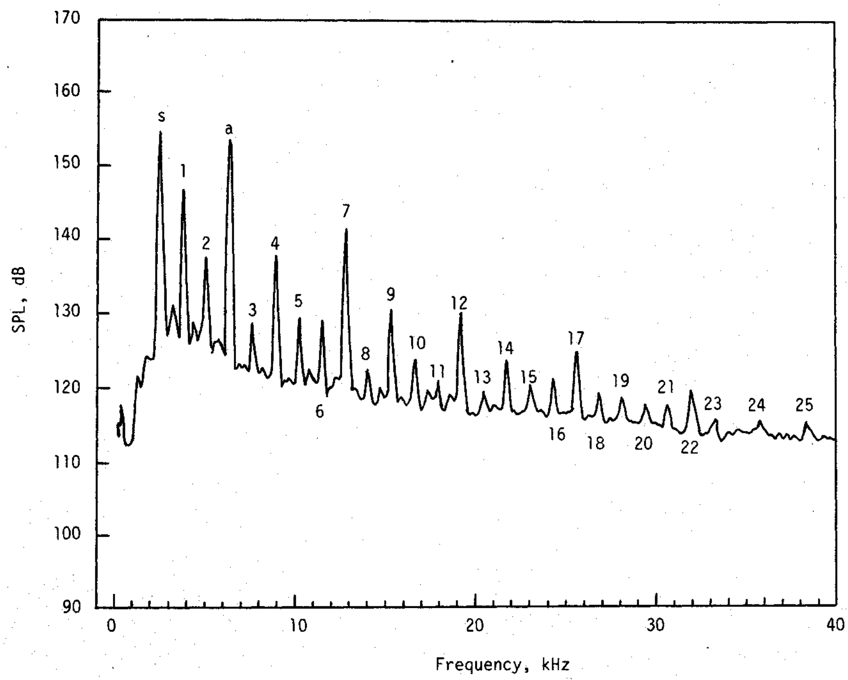

Fig. 2 The acoustic spectrum of a supersonic rectangular impinging jet; nozzle dimensions 2.013 × 0.475 in. M_j = 1.29, L = 5.25 in. Frequencies of the combination tones are 1 = $a - s$, 2 = $2s$, 3 = $3s$, 4 = $a + s$, 5 = $2a - s$, 6 = $a + 2s$, 7 = $2a$, 8 = $3a - 2s$, 9 = $2a + s$, 10 = $3a - s$, 11 = $2a + 2s$, 12 = $3a$, 13 = $4a - 2s$, 14 = $3a + s$, 15 = $4a - s$, 16 = $3a + 2s$, 17 = $4a$, 18 = $3a + 3s$, 19 = $4a + s$, 20 = $5a - s$, 21 = $4a + 2s$, 22 = $5a$, 23 = $4a + 3s$, 24 = $6a - s$, 25 = $6a$; s = 2.5 kHz, a = 6.4 kHz.

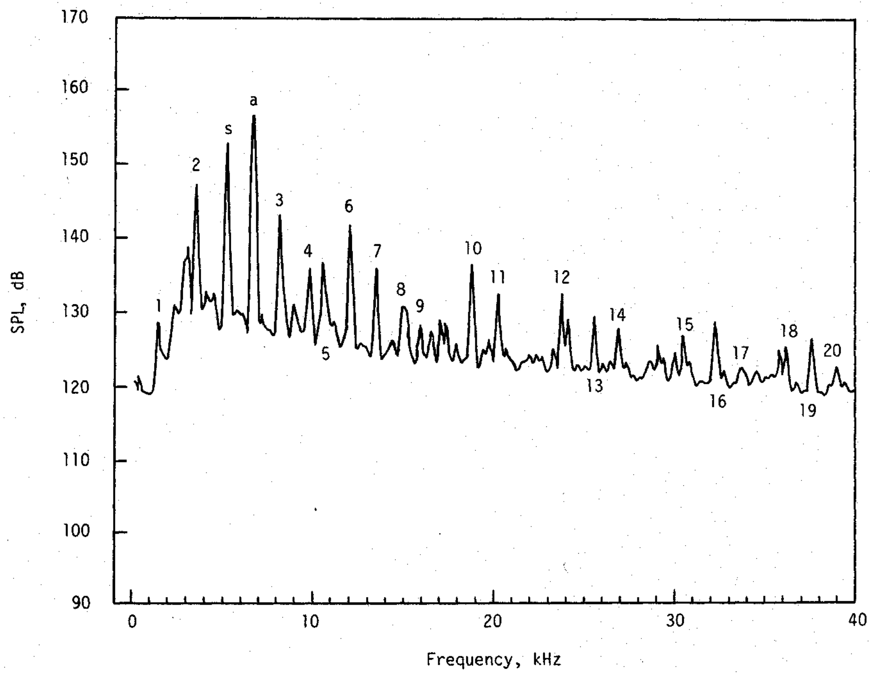

Fig. 3 The acoustic spectrum of a supersonic rectangular impinging jet; M_j = 1.29, L = 3.15 in. Frequencies of the combination tones are 1 = $a - s$, 2 = $2s - a$, 3 = $2a - s$, 4 = $3a - 2s$, 5 = $2s$, 6 = $a + s$, 7 = $2a$, 8 = $3a - s$, 9 = $3s$, 10 = $2a + s$, 11 = $3a$, 12 = $2a + 2s$, 13 = $3a + s$, 14 = $4a$, 15 = $3a + 2s$, 16 = $4a + s$, 17 = $5a$, 18 = $3a + 3s$, 19 = $4a + 2s$, 20 = $5a + s$; s = 5.3 kHz, a = 6.7 kHz.

end of the spectrum is formed primarily by the first few difference tones.

The jet facility used in the experiment has an internal tone at a very low frequency (0.6 kHz). This frequency appears in a large number of measured spectra independent of jet Mach number and nozzle-to-wall distance and hence can be readily identified. This tone appears as the lowest frequency peak (labeled e) in the spectrum of Fig. 4. Because of nonlinearity, this tone and the two basic frequencies also form combination tones. Since frequency e is small, these combination tones usually appear as sidebands of the harmonics and combination tones of the two dominant basic tones. In Fig. 4, 28 tones other than e, s, and a are labeled. Their frequency relationships to s and a and the facility tone e are given in the legend of the figure.

The combination tones are generated by the nonlinear oscillatory motions of the jet and their interaction with the wall. Figure 7 is an example of the motion of the jet at a combination tone frequency. The motion is highly complicated. It is important to point out that, although some of the combination

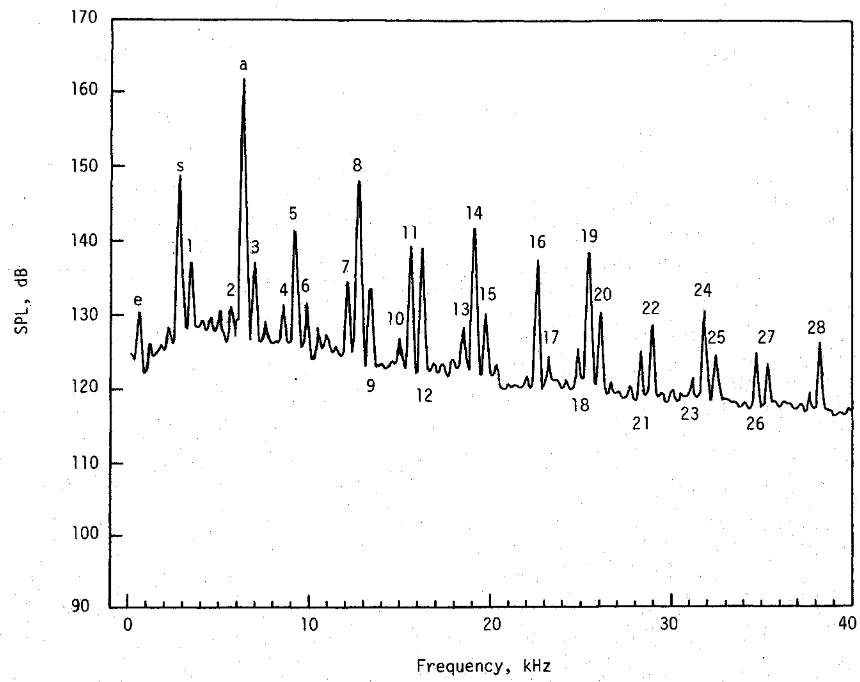

Fig. 4 The acoustic spectrum of a supersonic rectangular impinging jet; $M_j = 1.29$, $L = 2.55$ in. Frequencies of the combination tones are $1 = s + e$, $2 = a - e$, $3 = a + e$, $4 = s + a - e$, $5 = s + a$, $6 = s + a + e$, $7 = 2a - e$, $8 = 2a$, $9 = 2a + e$, $10 = 2a + s - e$, $11 = 2a + s$, $12 = 2a + s + e$, $13 = 3a - e$, $14 = 3a$, $15 = 3a + e$, $16 = 4a - s$, $17 = 3a + s$, $18 = 4a - e$, $19 = 4a$, $20 = 4a + e$, $21 = 5a - s - e$, $22 = 5a - s$, $23 = 5a - e$, $24 = 5a$, $25 = 5a + e$, $26 = 6a - s - e$, $27 = 6a - s$, $28 = 6a$; $e = 0.6$ kHz, $s = 2.9$ kHz, $a = 6.4$ kHz.

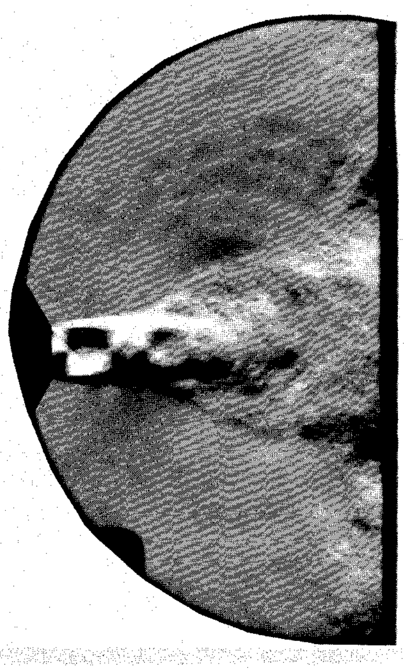

Fig. 5a Schlieren picture showing a supersonic impinging jet undergoing symmetric feedback instability; $M_j = 1.29$, $L = 3.45$ in., $f = 3.4$ kHz.

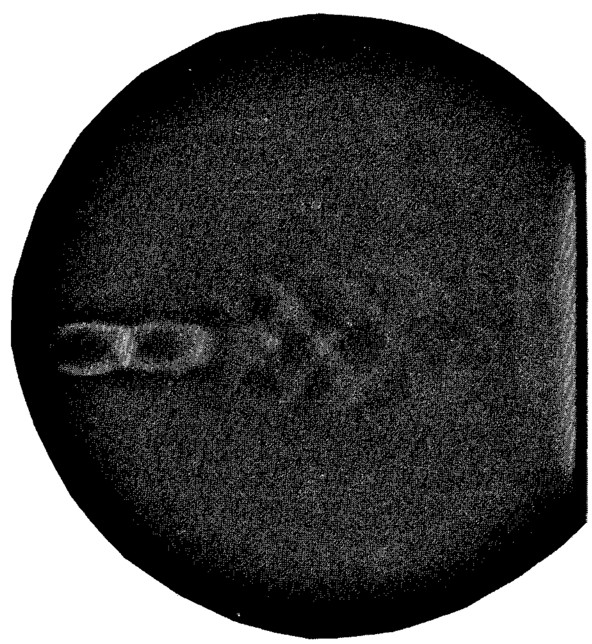

Fig. 5b Shadowgraph picture.

tone frequencies fall in the unstable range of the jet, the instability waves of the jet at these frequencies are not the primary factor responsible for the observed motions. The observed motions are driven principally by flow nonlinearities.

III. Feedback Acoustic Waves

As discussed in the last section, an impinging supersonic rectangular jet basically undergoes two types of feedback instabilities. Hydrodynamic stability theory when applied to two-dimensional jets (a reasonable approximation for jets with a large aspect ratio) reveals that these jets are subjected to two modes of instabilities. They are often referred to as the varicose (symmetric) and the sinuous (antisymmetric) instability modes. Thus the observed oscillations of the jet (see Figs. 4 and 5) are but the intrinsic symmetric and antisymmetric instabilities of the jet flow. At this point several observed facts appear to require an explanation. First is the consistency of the antisymmetric feedback instability mode having a higher frequency than the symmetric mode. Second is the monotonic decrease (except for the discrete jumps due to staging) of the two basic feedback frequencies as jet Mach number increases. To provide answers to these observations and for the purpose of developing a way to predict approximately the impingement

Fig. 6a Schlieren picture showing a supersonic impinging jet undergoing antisymmetric feedback instability; M_j = 1.29, L = 3.45 in., f = 6.3 kHz.

Fig. 7 Schlieren picture showing a supersonic impinging jet oscillating at a combination tone frequency; M_j = 1.29, L = 3.45 in., f = 9.7 kHz.

Fig. 6b Shadowgraph picture.

tone frequencies, it is necessary to examine the feedback loop more closely and quantitatively. Here we will adopt the feedback loop proposed by Tam and Ahuja.[10] Their model is essentially the same as other feedback models except that the feedback acoustic waves are taken to be the neutral upstream propagating wave modes of the jet flow. We believe, as in the work of Tam and Ahuja, that some of the characteristics of the feedback loop and hence those of the impingement tones are controlled by the characteristics of the feedback acoustic wave modes. These important characteristics of the feedback acoustic waves will now be determined by using a simple analytical jet model.

Consider a two-dimensional jet of thickness H, velocity u_j, and Mach number M_j bounded by two vortex sheets as shown in Fig. 8. Let p_0 and p_i be the pressure associated with the disturbances outside and inside the jet and $\zeta(x,t)$ the vertical displacement of the vortex sheets. By starting from the linearized equations of motion of a compressible inviscid flow, it is straightforward to show that the governing equations and boundary conditions for p_0, p_i, and ζ are

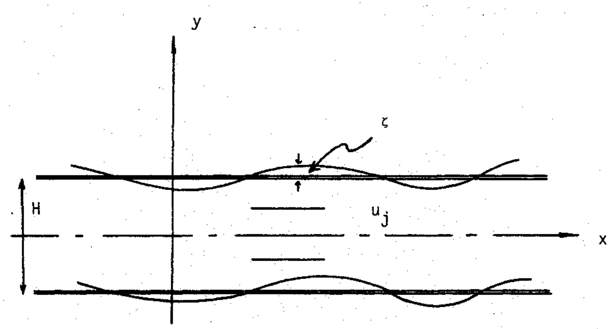

Fig. 8 Two-dimensional supersonic jet bounded by vortex sheets.

$$\frac{\partial^2 p_0}{\partial t^2} - a_\infty^2 \nabla^2 p_0 = 0, \qquad H/2 \le y \quad \text{or} \quad y \le -H/2 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x}\right)^2 p_i - a_j^2 \nabla^2 p_i = 0, \qquad -H/2 \le y \le H/2 \tag{3}$$

At $y = H/2$

$$p_0 = p_i \tag{4}$$

$$\frac{\partial^2 \zeta}{\partial t^2} = -\frac{1}{\rho_\infty}\frac{\partial p_0}{\partial y} \tag{5}$$

$$\left(\frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x}\right)^2 \zeta = -\frac{1}{\rho_j}\frac{\partial p_i}{\partial y} \tag{6}$$

At $y = 0$, for antisymmetric modes

$$p_i = 0 \tag{7}$$

and for symmetric modes

$$\frac{\partial p_i}{\partial y} = 0 \tag{8}$$

At $y \to \infty$,

$$p_0 \tag{9}$$

satisfies the outgoing wave or boundedness condition where a_∞, a_j, ρ_∞, and ρ_j are the speed of sound and gas density outside and inside the two-dimensional jet.

Now let us look for propagating wave solutions of the form

$$\begin{bmatrix} p_0(x, y, t) \\ p_i(x, y, t) \\ \zeta(x, t) \end{bmatrix} = \begin{bmatrix} \hat{p}_0(y) \\ \hat{p}_i(y) \\ \hat{\zeta} \end{bmatrix} e^{i(kx-\omega t)} \tag{10}$$

where k, the wave number, and $\omega(\omega > 0)$, the angular frequency, are as yet unspecified parameters. Substitution of Eq. (10) into Eqs. (2–9) and upon eliminating $\hat{\zeta}$, it is easy to find that $\hat{p}_0$ and $\hat{p}_i$ are given by the solution of the following eigenvalue problem:

$$\frac{d^2\hat{p}_0}{dy^2} - (k^2 - \omega^2/a_\infty^2)\hat{p}_0 = 0 \tag{11}$$

$$\frac{d^2\hat{p}_i}{dy^2} + [(\omega - u_j k)^2/a_j^2 - k^2]\hat{p}_i = 0 \tag{12}$$

At $y = H/2$

$$\hat{p}_i = \hat{p}_0 \tag{13}$$

$$\frac{1}{\rho_j(\omega - u_j k)^2}\frac{d\hat{p}_i}{dy} = \frac{1}{\rho_\infty \omega^2}\frac{d\hat{p}_0}{dy} \tag{14}$$

At $y = 0$, for antisymmetric modes

$$\hat{p}_i = 0 \tag{15}$$

and for symmetric modes

$$\frac{d\hat{p}_i}{dy} = 0 \tag{16}$$

Let us first consider the antisymmetric modes. The solution of Eq. (12) that satisfies the antisymmetric condition (15) is

$$\hat{p}_i = A \sin\{[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2} y\} \tag{17}$$

The solution of Eq. (11) that satisfies the outgoing wave or boundedness condition as $y \to \infty$ is

$$\hat{p}_0 = B \exp[-(k^2 - \omega^2/a_0^2)^{1/2} y] \tag{18}$$

In solutions (17) and (18), A and B are arbitrary constants. Substitution of these solutions into boundary conditions (13) and (14), the condition for nontrivial solution, gives the following dispersion relation for the parameters k and ω:

$$\frac{[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2}\rho_\infty\omega^2}{(k^2 - \omega^2/a_\infty^2)^{1/2}\rho_j(\omega - u_j k)^2} + \tan\{[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2} H/2\} = 0 \tag{19}$$

Similarly, it is straightforward to find that the dispersion relation for the symmetric modes is

$$\frac{[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2}\rho_\infty\omega^2}{(k^2 - \omega^2/a_\infty^2)^{1/2}\rho_j(\omega - u_j k)^2} - \cot\{[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2} H/2\} = 0 \tag{20}$$

Dispersion relations (19) and (20) may be used to calculate the instability waves of the two-dimensional jet flow. However, these dispersion relations as formulated are completely general and should describe all of the other possible intrinsic wave modes of the jet as well. Recently Tam and Hu[11] carried out an in-depth investigation of the wave modes of high-speed compressible circular jets using a similar vortex sheet jet model. They found two additional families of wave solutions beside the familiar Kelvin-Helmholtz instability waves. One of the families of waves consists of neutral waves with real k and ω. These waves are neither unstable nor damped, behaving very much like acoustic waves. A subset of these neutral waves (those with negative group velocity) propagates upstream by attaching themselves to the supersonic jet. Subsequently, Tam and Ahuja[10] suggested that these are the feedback acoustic waves of impinging jets. Here we will follow their suggestion. It will be shown that the observed characteristics of the impingement tones are controlled by the characteristics of these upstream propagating acoustic wave modes.

Figures 9 and 10 show typical dispersion relations of the neutral acoustic wave modes at a jet Mach number of 1.29 (the total temperature of the jet has been assumed to be the same as the ambient temperature, i.e., cold jet). The dispersion relations of the symmetric modes are given in Fig. 9 whereas those of the antisymmetric modes are given in Fig. 10. The upstream propagating wave modes have negative group velocity ($d\omega/dk < 0$) or negative slope in the $\omega - k$ plane. They are shown as the solid portion of the curves in Figs. 9 and 10. The allowable frequencies of these curves are limited to certain bands. The upper limit of each band is defined by the maximum point of the dispersion curve of the particular mode. The lower limit lies on the straight line

$$k^2 - \omega^2/a_\infty^2 = 0 \tag{21}$$

On this line in the ω-k plane, the first term of Eqs. (19) and (20) becomes infinite. To satisfy the dispersion relation, therefore, the second term must also tend to infinity. This requires the argument of the tangent (or cotangent) function to be equal to an integer plus a half times π (integer times π). That is,

Antisymmetric modes:

$$[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2} H/2 = (n - 1/2)\pi \tag{22}$$

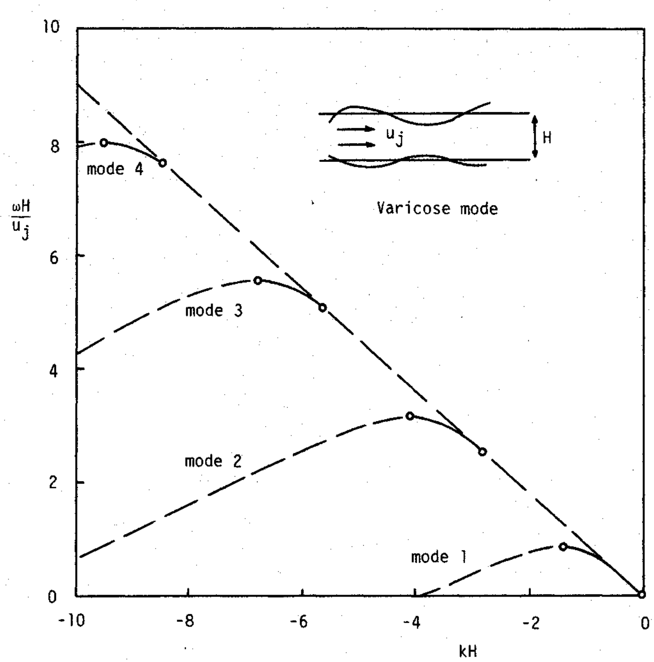

Fig. 9 Dispersion relations of symmetric upstream propagating acoustic wave modes of a Mach 1.29 two-dimensional (cold) jet, ○——○.

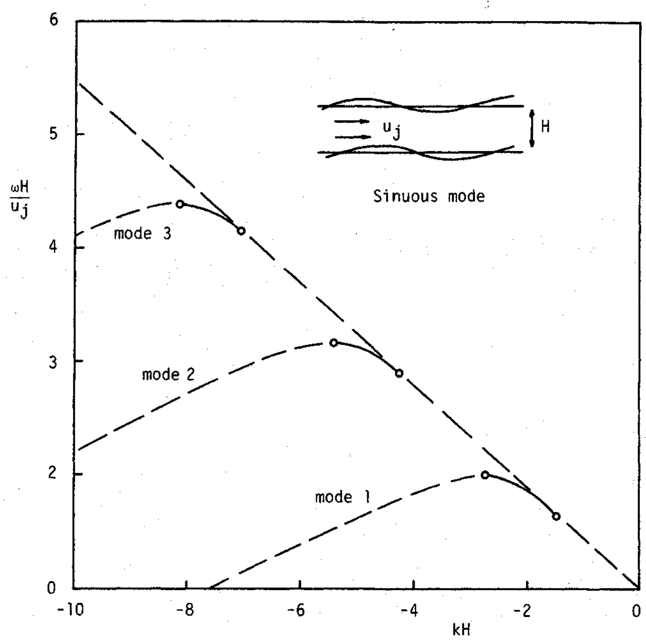

Fig. 10 Dispersion relations of antisymmetric upstream propagating acoustic wave modes of a Mach 1.29 two-dimensional (cold) jet, ○———○.

Symmetric modes:

$$[(\omega - u_j k)^2/a_j^2 - k^2]^{1/2} H/2 = (n - 1)\pi \qquad (23)$$

where $n = 1, 2, 3, \ldots$. The simultaneous solutions of Eqs. (21) and (22) or (21) and (23) provide the lower limits of the frequency bands at a given jet Mach number and jet to ambient temperature ratio.

It is to be noted that the lowest frequency band of the symmetric mode has frequencies lower than that of the antisymmetric modes. Also it is well known that the instability waves of the jet, the same waves that drive the impingement feedback loops, exist only at low Strouhal number. They will not match the frequency of the higher order symmetric or antisymmetric upstream propagating neutral wave modes to form feedback loops. This frequency matching condition, therefore, suggests that the impingement tone frequency associated with the antisymmetric mode must be higher than that of the symmetric mode for a given jet operating condition. This is in complete agreement with experimental observation.

Now if indeed the feedback is achieved by the upstream propagating neutral waves, then the frequencies of the impingement tones must lie within the first allowable frequency band as predicted by the dispersion relations. These frequencies, of course, must satisfy the feedback condition of Eq. (1). However, the band restriction can be satisfied by an appropriate choice of the as yet unspecified integer value n. The bandwidth is large enough to allow an abrupt change in the value of n. Figure 11 shows the allowable frequency band of the symmetric and the antisymmetric modes for cold jets as functions of jet Mach number. The average frequency of the two bands decreases with increase in Mach number, implying the same must be true for the impingement tone frequencies. Plotted in this figure also are the observed two basic impingement tone frequencies at seven jet Mach numbers measured by Norum.[9] At each jet operating condition, the nozzle-to-wall distance L is gradually increased. The impingement tone frequencies vary as L/H changes. The aggregated frequencies are shown as black bands in Fig. 11. Occasionally, isolated frequencies are found. They are shown as isolated points in this figure. As can be seen, all of the measured tone frequencies lie within (or almost within) the calculated allowable frequency bands. It is believed that this agreement between the calculated results and experiments offers strong evidence that the feedback acoustic waves are actually made up of the upstream propagating acoustic wave modes of the jet flow. Further, the frequency band of the first mode of these waves, both symmetric and antisymmetric, may be used as a reasonable first estimate of the impingement tone frequencies.

IV. Summary

The noise spectrum generated by a supersonic rectangular jet of large aspect ratio impinging on a wall is dominated by a multitude of tones. In many cases, 20 or more spectral peaks can be readily identified. A careful analysis of the frequency structure of these tones indicates that there are two basic tone frequencies. One is associated with a symmetric feedback instability mode. The other is associated with an antisymmetric mode. The frequency of the antisymmetric mode is higher than that of the symmetric mode. All of the other tones are com-

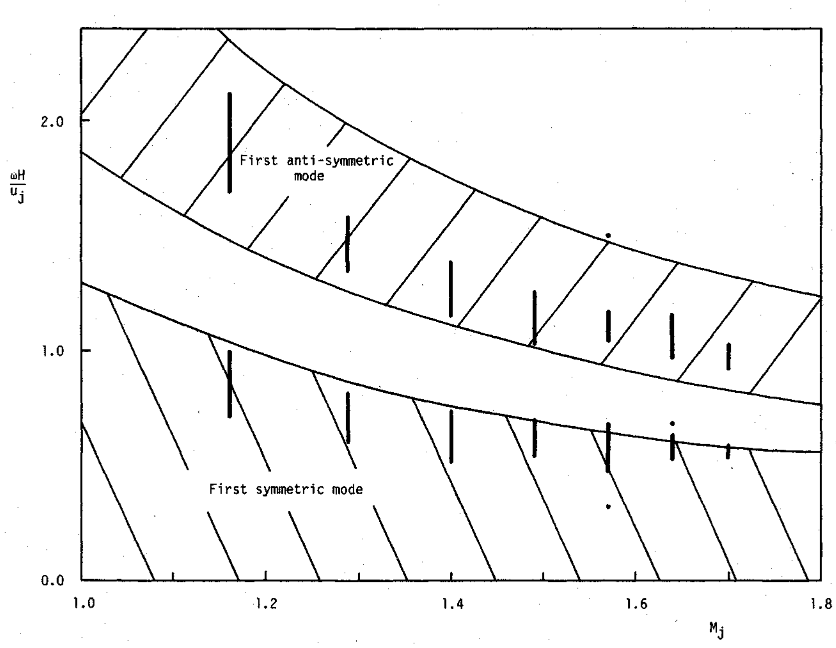

Fig. 11 Allowable frequency band of the first symmetric and antisymmetric upstream propagating acoustic modes of two-dimensional supersonic cold jets. Theoretical (shaded); I, measured impinging tone frequencies; ●, one data point.

bination tones of the two basic tones and their harmonics. That is, their frequencies are equal to the sums and differences of the frequencies of the two basic tones and their harmonics. These combination tones are generated by the nonlinearities of the jet flow. The experimental facility may sometimes produce a low frequency internal tone independent of jet operating conditions. The presence of such an internal tone introduces further combination tones usually in the form of sidebands to the combination tones of the two basic frequencies.

On following the work of Tam and Ahuja,[10] it is proposed that for supersonic rectangular impinging jets of large aspect ratio the feedback acoustic waves from the wall to the nozzle lip are made up of the upstream propagating neutral acoustic modes of the jet flow. The characteristics of these modes are studied using a vortex sheet jet model. The theory reveals that there are two types of waves, the symmetric and the antisymmetric waves. For each type of wave, there are infinitely many modes. The allowable frequencies of each mode are confined to a narrow band. Only the first mode of the symmetric and antisymmetric waves have frequencies that can match those of the jet instability waves. Under a given jet operating condition, the frequency of the antisymmetric first feedback acoustic mode is always higher than that of the symmetric mode, implying the same for the impingement tones. It is also found that the measured basic impingement tone frequencies do fall in the theoretical allowable frequency bands over a wide range of jet Mach number. This agreement between theory and experiment provides strong support for the validity of the feedback model of Tam and Ahuja.

Acknowledgment

The work of the first author was supported by NASA Langley Research Center, Grant NAG1-421.

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